Various 2-D Kinematics and Dynamics

2-D Relative Motion

Example #1
A boat wants to move straight across a 30.0 m wide river. The current in the river is moving at 4.0 m/s and the boat's engine speed is 6.0 m/s. At what angle should the boat head?

\[
\sin \theta = \frac{4.0 \text{ m/s}}{6.0 \text{ m/s}}
\]

\[
\theta = \sin^{-1} \left( \frac{4.0}{6.0} \right)
\]

\[
\theta = 42^\circ
\]

The boat should head upstream @ 42°

Example #2
A boat heads directly across a 50.0 m wide river at 2.00 m/s. The current is 1.00 m/s.

a) find the resultant speed and direction

\[
U_r = \sqrt{2.0^2 + 1.0^2} = \sqrt{5} \text{ m/s}
\]

\[
\theta = \tan^{-1} \left( \frac{1.0}{2.0} \right)
\]

\[
\theta = 26.6^\circ
\]

\[
U_r = 2.24 \text{ m/s @ 26.6° downstream}
\]

b) find the resultant distance the boat travels in crossing the river

\[
\cos 26.6^\circ = \frac{50\text{ m}}{d_r}
\]

\[
d_r = \frac{50\text{ m}}{\cos 26.6^\circ}
\]

\[
d_r = 55.9\text{ m}
\]
Example #3
A plane flies West at 300 kph. The wind blows NE at 100 kph. What is the resultant motion of the plane relative to the ground? Draw and fill in the chart provided to solve by components.

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane = 300 West (V₁)</td>
<td>-300</td>
<td>0</td>
</tr>
<tr>
<td>Wind = 100 NE (V₂)</td>
<td>+100 Cos 45</td>
<td>+100 Sin 45</td>
</tr>
<tr>
<td>Result (Vᵣ)</td>
<td>-229.2893...</td>
<td>70.7106...</td>
</tr>
</tbody>
</table>

We can also solve this question using the sine and cosine law:
NOTE: In the previous question, we were asked to add the planes vector ($V_1$) and the wind vector ($V_2$) to produce a resultant vector ($V_r$) 

\[ V_1 + V_2 = V_r \]

However, if we were to solve for the wind, we would do the following \[ V_r - V_1 = V_2 \]

2-D Dynamic Problems

Example #4

Find normal force, friction and acceleration

\begin{align*}
\text{200. N} & \\
\text{20.0°} & \\
\mu &= 0.100 \\
\end{align*}

\[ F_N = (552 \text{ N}) \]

\[ F_f = (490 \text{ N}) \]

\[ F_{\text{total}} = 200 \cos 20° \]

\[ a = \frac{2.6 \text{ m/s}^2}{50} \]

\[ a = 0.052 \text{ m/s}^2 \]
Example #5
Find the acceleration of the 100 kg boat below (note: top view, so mg is into the page and buoyant force from water is out of page) using a component solution.

\[ T_1 = 150 \cos 20^\circ \quad T_2 = 300 \cos 20^\circ \]
\[ T_1 = 150 \sin 20^\circ \quad T_2 = 300 \sin 20^\circ \]

\[ EF_x = 400 - 76 \quad EF_y = -98.6969 \]

\[ F_c = \sqrt{(4.0 \text{ m/s}^2)^2 + (-9.9 \text{ m/s}^2)^2} = 10.12 \text{ m/s}^2 \]

**Example 4:** In example 3, draw the net force vector as the sum of the two force vectors.
Example #6 – Time permits
Find the tension in both cables

\[ T_1 \cos 35^\circ = T_2 \sin 35^\circ \]

\[ T_1 = \frac{F_g}{\cos 35^\circ} = \frac{98 N}{0.819} = 117.4 N \]

\[ T_2 = \frac{F_g \sin 35^\circ}{\sin 35^\circ} = \frac{98 N \times 0.573}{0.819} = 68.62 N \]