Example #1
Name the following vectors

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>1. 50°</td>
<td>2. 30°</td>
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<tr>
<td>3. 42°</td>
<td>4. 37°</td>
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<tr>
<td>5. 27°</td>
<td>6. 25°</td>
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Example #2 – Vector Components
Find the components of a vector that has a velocity of 20 m/s at 30° N of W
Example #3 – Adding Vectors
Add the following vectors

a) 15 m East and 25 m North

\[ r = \sqrt{(15)^2 + (25)^2} \]
\[ r = 29.1547 \text{ m} \]
\[ \theta = \tan^{-1} \left( \frac{25}{15} \right) \]
\[ \theta = 57.038^\circ \]

b) Given \( r_1 = 3.0 \text{ m} \) NE, \( r_2 = 2.0 \text{ m}, 30^\circ \) S of E, draw/measure \( r_1 + r_2 \)

1) Solve using components

\[ x_1 = \frac{3 \sqrt{2}}{2} \text{ m} \]
\[ x_2 = \frac{1.2}{2} \text{ m} \]
\[ x_1 + x_2 = \frac{3 \sqrt{2} + 1.2}{2} \text{ m} \]
\[ y_1 = \frac{3 \sqrt{2}}{2} \text{ m} \]
\[ y_2 = 2 \text{ m} \]
\[ y_1 + y_2 = \frac{3 \sqrt{2} + 2}{2} \text{ m} \]

\[ r_1 = \sqrt{3.85337^2 + 1.1232^2} \]
\[ r_1 = \sqrt{16.108827} \]
\[ r_1 = 4.01 \text{ m} \]

\[ \theta = \tan^{-1} \left( \frac{1.1232}{3.85337} \right) \]
\[ \theta = 16^\circ \text{ N of E} \]
2) Solve not using components

c) An object is initially moving at 3.0 m/s East. It accelerates South at 1.5 m/s² for 4.0 seconds. Find the final velocity by adding: 

\[ v_f = v_i + a\Delta t \]

\[ v_f = v_i + a\Delta t \]
**Example #4 – Subtracting Vectors**

Subtract the following vectors:

a) For the vectors A and B find A - B by drawing A + (-B)

b) Given \( v_f = 2.00\text{m/s N} \), \( v_i = 3.00\text{ m/s E} \) and elapsed time \( \Delta t = 3.00 \text{ sec} \)

   a. draw/measure: \( v_f - v_i \)
   b. find the acceleration vector \( a = (v_f - v_i) / \Delta t \)

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**Note** the icon \( \hat{i} \) (i hat) is for x components, \( \hat{j} \) (j hat) is for y components.
Question 2, 4, 6, 8

Exercises

1. Given \( v = 3.0 \text{ m/s} \) at 20° S of E draw and give the measure of
   a) 2 \( v \)
   b) \(-v\)  
   (6.0 m/s at 20° SofE; 3 m/s at 20° NofW)

2. If \( x = 2.00 \text{ N SW} \) and \( y = 3.00 \text{ N 10.0° N of W} \) then find:
   a) \( x + y \)
   b) \( 2x + y \)
   c) \( y - x \)
   d) \( 2x - y \)
   (4.46 N @ 11.6° SofW; 6.23 N @ 21.8° SofW; 2.47 N @ 51.5° NofW; 3.35 N @ 87.8° SofE)

3. Given:
   \( A = 3.00 \text{ cm at 60.0° N of E} \)
   \( B = 7.00 \text{ cm at 40.0° W of N} \)
   \( C = 6.00 \text{ cm at 60.0° S of E} \)
   \( D = 4.00 \text{ cm W} \)
   Draw and measure the sum: \( A + B + C + D \)  
   (4.86 cm at 34.6° N of W)

4. Add the vectors:
   \( d_1 = 6.0 \text{ cm at 30.0° N of E} \)
   \( d_2 = 5.0 \text{ cm at 40.0° E of S} \)  
   (8.4 cm at 5.6° S of E)

5. Solve for \( A \) given that \( A + B = C \); \( B = 10.0 \text{ N 20.0° S of E} \); \( C = 8.0 \text{ N at 25° S of E} \)
   (2.1 N @ 1.0° N of W)

6. An object is moving at \( v_i = 3.0 \text{ m/s} \) East. It accelerates for 2.0 seconds at \( a = 2.0 \text{ m/s}^2 \) North. a) find \( v_f \) by adding \( v_i \) and at  
   b) why is \( v_f \) not 7.0 m/s?

7. Given \( v_i = 40.0 \text{ m/s E} \); \( v_f = 30.0 \text{ m/s S} \); \( t = 5.0 \text{ s} \)
   a) find  
   \[ a = \frac{(v_f - v_i)}{t} \]
   b) why is \( a \) not -2 m/s²?
   (10. m/s² at 38° S of W)

8. Add boat/plane velocity to wind/water velocity to find resultant velocity.
   \[ V_{boat} + W = V_{result} \]
   a) A boat has an engine speed of 7.00 m/s North. The water current is flowing at 5.00 m/s West. Find the resulting velocity. (8.60 m/s at 35.5° W of N)
   b) A boat that moves at 5.00 m/s in still water is heading South. It encounters a current of 2.00 m/s West. Find the resulting velocity. (5.39 m/s 21.8° W of S)
   c) A plane has an airspeed of 300. kph NE. The wind is blowing at 50.0 kph S. Find the resulting velocity. (267 kph at 37.4° NofE)
   d) A plane is heading at 450. kph 25.0° E of N. The wind is blowing at 70.0 kph E. Find the resulting velocity. (483 kph at 57.5° N of E)

9. A mass is thrown straight up at 10. m/s. At what times will it be at height 5.0 m off the ground?  
   (0.88 s; 1.2 s)
Exercises

1. Convert to components
   a) 50.0 m/s at 30.0° N of E (43.3î + 25.0j)
   b) 25.0 m/s, 40.0° S of E (19.2î - 16.1j)
   c) 75.0 m at 40.0° S of W (-57.4î - 48.2j)

2. Convert to polar the following velocities of a projectile
   (39.0 m/s, 50.2° above hzntl
   25.0 m/s hzntl
   39.0 m/s, 50.2° below hzntl
   51.5 m/s, 60.9° below hzntl)

3. Given A = 20.0 m, 30.0° N of E
   B = 10.0 m, 20.0° W of S
   a) convert both vectors to rectangular components
   b) add the vectors in rectangular
   c) convert the answer to polar form
   d) make a rough scale drawing of A + B to confirm your answer to c)
      (17.3î + 10.0î; -3.42î - 9.40j; 13.9î + 0.6j; 13.9 at 2.48° NoFSE)

4. Given \( \mathbf{v}_i = 10.\hat{i} + 20.\hat{j} \) m/s \( \mathbf{a} = -10.\hat{j} \) m/s² \( t = 3.0 \) s, find in components
   a) the vector at \( t \)
   b) the vector \( \mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \)
      (-30.\hat{j}; 10.\hat{i} - 10.\hat{j})

5. Given \( \mathbf{v}_i = 40.\text{m/s S} \) \( \mathbf{v}_f = 30.\text{m/s W} \) \( t = 5.0 \) s
   a) write both vectors in rectangular
   b) find \( \mathbf{v}_f - \mathbf{v}_i \) in rectangular
   c) find \( \mathbf{a} \) in rectangular
   d) change to polar \( \mathbf{a} \)
      (-40.\hat{j}; -30.\hat{i}; -30.\hat{i} + 40.\hat{j}; -6.0î + 8.0j; 10. m/s² 53° N of W)
6. Add boat/plane velocity $V_{\text{boat}}$ to wind/water velocity $W$ to find resultant velocity $V_{\text{result}}$.
   a) A plane travels southeast at 400 kph. The wind is blowing west at 80.0 kph. Find the plane's velocity relative to the ground.
   b) A plane heads 40.° W of N at an airspeed of 380 kph. The plane is seen to move at 10.° W of N at 350 kph. What is the wind velocity?
      a) $(283\hat{i} - 283\hat{j}) + (-80\hat{i}+0\hat{j}) = 203\hat{i} -283\hat{j} = 348$ kph at 54.4° S of E
      b) $(-60.8\hat{i}+345\hat{j}) - (-244\hat{i}+291\hat{j}) = 183\hat{i} + 54\hat{j} = 190$ kph at 16° N of E

7. Fred and Wilma leave their house at the same time heading for the bus stop 100. m away. Fred starts at rest, accelerates at 0.500 m/s² up to a maximum speed of 5.00 m/s, which he holds as he runs for the bus stop. Wilma gives Fred a 5.00 second head start and then accelerates for 6.00 seconds up to her maximum speed of 6.00 m/s. Who gets to the bus stop first, and how long do they wait for the second person to arrive? (Wilma takes 24.7s; Fred takes 25.0 s)

8. A car starts at rest and accelerates in 7.00 seconds up to a maximum speed of 30.0 m/s, and holds this speed for another 25.0 seconds.
   a) find the total distance travelled
      (855 m)
   b) find the average speed
      (26.7 m/s)

9. Make a graph of x-t and v-t for the car in question #8

    
    $A = 100. \text{m}, 40.\text{o} \text{N of W}$
    $B = 80. \text{m}, 10.\text{o} \text{E of S}$
    (64 m at 13° S of W; 170 m at 58° N of W)

11. Consider the following vectors
    $A = 80.0 \text{m}, 20.0\text{o} \text{N of W}$
    $B = 55.0 \text{m}, \text{SW}$
    a) find their components
       $(-75.2\hat{i} + 27.4 \hat{j}; -38.9\hat{i} - 38.9 \hat{j})$
    b) find $A + B$
       $(115. \text{m}, 5.77\text{o} \text{S of W})$
    c) find $A - B$
       $(75.5 \text{m}; 61.3\text{o} \text{N of W})$